

DYNAMICS OF A ROTOR IN FILM LUBRICATION BEARINGS

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A dynamic model of a rotor system with hydrodynamic plain bearings in which account has been taken of the actually existing phenomenon of the circular anisotropy of the rigidity of hydrodynamic plain bearings has been proposed. The operation of the rotor system has been modeled with the use of uniformly distributed sequences and the region of states of the system representing a set of instantaneous positions of the center of the end rotor surface by which the accuracy of its rotation is evaluated has been obtained.

Supports with hydrodynamic and hydro- and aerostatic plain bearings have gained wide acceptance in different technological systems. These are metal-cutting machine tools, generating equipment for electric power stations, pumps of different types, centrifuges, etc. The precision longevity and normal operating period of different types of rotor equipment are largely determined by the precision and the operating period of plain bearings (supports).

The necessity of improving the precision characteristics of bearings and extending their operating period is dictated not only by the increasingly more accelerating growth in the level of requirements imposed on the quality and competitiveness of newly created machines and mechanisms but also by the current tendency of transition to establishing "service life by their state." This is due to the fact that it has become necessary at present to extend the operating period of functioning equipment that has already exhausted or is exhausting its design service life. The efficiency of solution of these problems depends on the correctness of the evaluation of the state of the equipment, which usually represents a multiparametric system with several figures of merit.

Hydrodynamic bearings possessing a number of substantial advantages as compared to other kinds of bearings are frequently the most efficient (and sometimes the only) technical solution ensuring the required characteristics of rotor systems [1]. Figure 1 gives the factors influencing the precision of a rotor system with hydrodynamic plain bearings. All of them can be subdivided into external and internal factors. Among the external factors are the action of the external medium, which can be expressed in any foreign vibrational processes, for example, from nearby equipment, and in the thermal radiation from the equipment, the heating system of the shop, solar heat, etc., and the dynamic and thermal actions of the machine on which the rotor is mounted on the rotor itself.

The internal factors can be subdivided into structural-technological and operating factors. Among the first are, in particular, the architecture and structural dimensions of the rotor system, the initial lack of precision of the components of the bearings and the rotor itself, clearances established in assembling the unit, and the initial rigidity of the bearing's structure. Among the second are the rotational velocity of the rotor, the intrinsic heating of the rotor, for example, from rubbing in bearings, the viscosity of the working fluid in the bearing chamber and the viscosity change due to the heating of this chamber, the rigidity of the oil layer and the running clearance which is established in the bearing in rotation of the shaft, and the disbalance of the rotor as a unit and its change with wear of the supports. The initial disbalance of the rotor is, certainly, a structural-technological factor but it changes in operation, and this enables us to classify the disbalance with operating factors [2].

We consider a standard rotor unit (used in mechanical engineering) with a driving pulley on one end of the shaft and radial load on the other as a particular case of a rotor system. To investigate of the accuracy of the rotation of a rotor in hydrodynamic plain bearings let us consider the rotor unit as a dynamic system with elastic supports with viscous damping which has localized masses and is loaded by radial forces and forces from the rotary drive and from

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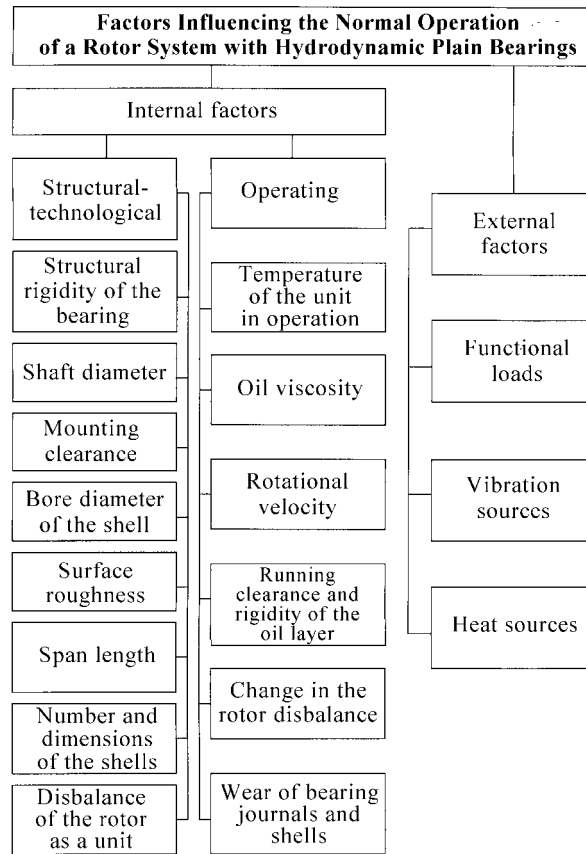


Fig. 1. System analysis of the factors influencing the operating period of a rotor unit.

the disbalance of rotating masses. The vibrations of such a system can be described by a system of differential equations.

In constructing the dynamic model, we have made the following assumptions:

(1) The rotor may be considered to be a perfectly rigid body, since the rigidities of its supports are several times lower than the flexural rigidity of the shaft itself; consequently, its elastic displacements are insignificant as compared to such caused by the pliability of the supports;

(2) The level of vibrations of the rotor in the case of the working rotational velocity is determined mainly by its disbalance;

(3) The rigidity of hydrodynamic plain bearings possesses circular anisotropy.

The design diagram of the rotor system, employed for construction of the dynamic model, is presented in Fig. 2a. The arrangement of segmentary shells in the supports and the direction of action of the forces is shown in Fig. 2b.

To compose the system of differential equations of motion of the rotor [3] we select the coordinate vector

$$\mathbf{z}^T = (z_1, z_2),$$

where $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, and the coordinates of displacement of the shaft in the first and second supports are as follows: (x_1, y_1) and (x_2, y_2) .

The equations of motion are

$$\frac{M}{l} (\ddot{z}_1 l_2 - \ddot{z}_2 l_1) + c_1 z_1 + c_2 z_2 + h_1 \dot{z}_1 + h_2 \dot{z}_2 = \omega^2 \exp(i\omega t) (m_1 \varepsilon_1 + m_2 \varepsilon_2 \exp(i\gamma)) + \mathbf{R} - i\mathbf{G} - i\mathbf{N}_2;$$

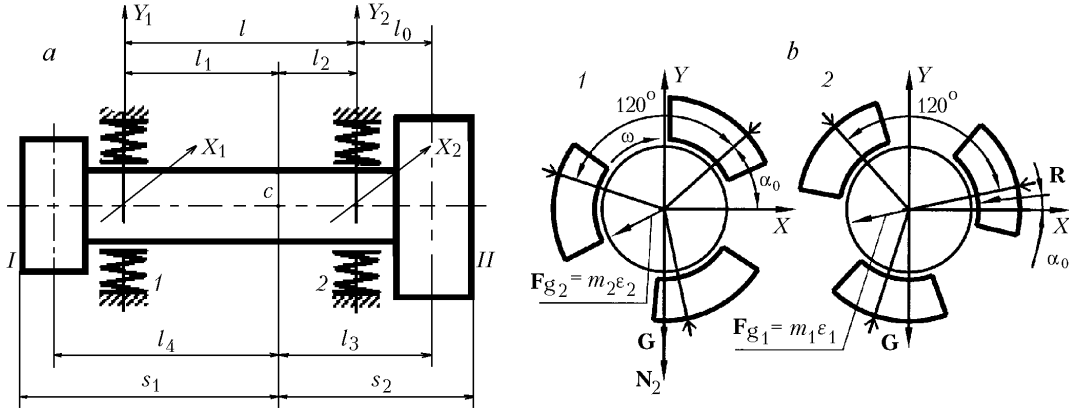


Fig. 2. Design diagram of the rotor system (a), arrangement of the segmentary shells, and directions of action of the forces (b).

$$\begin{aligned}
 & i \frac{I_p \omega}{l} (\dot{z}_1 - \dot{z}_2) + \frac{I_e}{l} (\ddot{z}_2 - \ddot{z}_1) + c_1 z_1 l_1 + c_2 z_2 l_2 + h_1 \dot{z}_1 l_1 + h_2 \dot{z}_2 l_2 = \\
 & = \omega^2 \exp(i\omega t) (m_1 \varepsilon_1 s_1 + m_2 \varepsilon_2 s_2 \exp(i\gamma)) + \mathbf{R} l_3 - i \mathbf{N}_2 l_4.
 \end{aligned} \tag{1}$$

We introduce the notation

$$c_1 z_1 = \mathbf{F}_1, \quad c_2 z_2 = \mathbf{F}_2, \quad h_1 \dot{z}_1 = \mathbf{H}_1, \quad h_2 \dot{z}_2 = \mathbf{H}_2,$$

taking into account that

$$\exp(i\omega t) \exp(i\gamma) = \cos(\omega t + \gamma) + i \sin(\omega t + \gamma),$$

we project Eq. (1) onto the coordinate axes X and Y . In this case the mathematical dynamic model will have the form

$$\begin{aligned}
 & \frac{M}{l} (\ddot{x}_1 l_2 - \ddot{x}_2 l_1) + F_{1x} + F_{2x} + H_{1x} + H_{2x} = \omega^2 (m_1 \varepsilon_1 \cos \omega t + m_2 \varepsilon_2 \cos(\omega t + \gamma)) - R \cos \varphi_0; \\
 & \frac{M}{l} (\ddot{y}_1 l_2 - \ddot{y}_2 l_1) + F_{1y} + F_{2y} + H_{1y} + H_{2y} = \omega^2 (m_1 \varepsilon_1 \sin \omega t + m_2 \varepsilon_2 \sin(\omega t + \gamma)) - R \sin \varphi_0 - G - N_2; \\
 & - \frac{I_p \omega}{l} (\dot{x}_1 - \dot{x}_2) + \frac{I_e}{l} (\ddot{x}_2 - \ddot{x}_1) + F_{1x} l_1 + F_{2x} l_2 + H_{1x} l_1 + H_{2x} l_2 = \\
 & = \omega^2 (m_1 \varepsilon_1 s_1 \cos \omega t + m_2 \varepsilon_2 s_2 \cos(\omega t + \gamma)) - R \cos \varphi_0 l_0; \\
 & - \frac{I_p \omega}{l} (\dot{y}_1 - \dot{y}_2) + \frac{I_e}{l} (\ddot{y}_2 - \ddot{y}_1) + F_{1y} l_1 + F_{2y} l_2 + H_{1y} l_1 + H_{2y} l_2 = \\
 & = \omega^2 (m_1 \varepsilon_1 s_1 \sin \omega t + m_2 \varepsilon_2 s_2 \sin(\omega t + \gamma)) - R \sin \varphi_0 l_3 - N_2 l_4.
 \end{aligned} \tag{2}$$

The rigidity of each plain bearing can be represented as the sum of two series-connected rigidities: the oil-wedge rigidity C_{oil} and a constant structural rigidity C_{str} . The total rigidity for each shell of a multiwedge support is determined by the formula [4]

$$c = \frac{C_{oil} C_{str}}{C_{oil} + C_{str}},$$

where C_{oil} can be found in terms of the force developing in the support and the shift of the center of the rotor under its action. For each shell of the bearing the force developing in the oil wedge can be calculated from the expression [4]

$$P = \frac{5.1 \cdot 10^{-2} \mu n D_{sh} B_{sh}^2 L C_L}{\delta^2 \left(1 - \frac{2e}{\delta} \cos \theta\right)^2}. \quad (3)$$

We introduce the notation $a = 5.1 \cdot 10^{-2} \mu n D_{sh} B_{sh}^2 L C_L$; then (3) is transformed to

$$P = \frac{a}{(\delta - 2e \cos \theta)^2}.$$

Taking into account that e is the value of the eccentricity of the center of the rotor shaft in the process of rotation and denoting the coordinates of the shaft's center at the instant of time t by $X = X(t)$ and $Y = Y(t)$, we obtain

$$e = \sqrt{x^2 + y^2}.$$

The value of the opening of the gap for each shell, as is seen in Fig. 2a, is $u = e \cos \theta$; then (3) can be transformed and written as

$$P = \frac{a}{(\delta - 2u)^2}.$$

In view of the symmetry of the three shells, we drop the constant component a/δ^2 in this expression; then P becomes equal to

$$P = \frac{a}{(\delta - 2u)^2} - \frac{a}{\delta^2} = \frac{4au(\delta - u)}{\delta^2(\delta - 2u)^2}.$$

Solving this equation for u , we find

$$|2u - \delta| = \frac{\delta \sqrt{a}}{\sqrt{a + P\delta^2}} \Rightarrow u = \frac{1}{2} \left(\delta + \frac{\delta \sqrt{a}}{\sqrt{a + P\delta^2}} \right).$$

If account is taken of the structural rigidity, the total shift of the shaft under the action of the resultants of external forces will be equal to

$$u = \frac{P}{C_{str}} + \frac{1}{2} \left(\delta + \frac{\delta \sqrt{a}}{\sqrt{a + P\delta^2}} \right).$$

Now we can determine the force $F(u)$ corresponding to the shift u of the bearing shell:

$$2F(u) + C_{str} \delta \left(1 + \frac{\sqrt{a}}{\sqrt{a + F(u)} \delta^2} \right) = 2C_{str} u.$$

We introduce the notation

$$W = \sqrt{\frac{a + F(u) \delta^2}{a \delta^2}} = \sqrt{\frac{1}{\delta^2} + \frac{F(u)}{a}},$$

whence

$$F(u) = \left(W^2 - \frac{1}{\delta^2} \right) a,$$

then

$$2W^2a - \frac{C_{\text{str}}}{W} + C_{\text{str}}(\delta - 2u) + \frac{2a}{\delta^2} = 0, \quad W^3 = W \frac{C_{\text{str}}\delta^2(\delta - 2u) - 2a}{2a\delta^2} + \frac{C_{\text{str}}}{2a} = 0.$$

There exists a unique solution of the equation of the form $W^3 + \rho W + q = 0$; it is prescribed by the formula

$$W = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{\rho}{3}\right)^3 + \left(\frac{q}{3}\right)^2}} - \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{\rho}{3}\right)^3 + \left(\frac{q}{3}\right)^2}}.$$

For the force $F(u)$ we obtain the expression

$$F(u) = \left[\left(\sqrt[3]{\frac{C_{\text{str}}}{4a} + \sqrt{\left(\frac{\rho}{3}\right)^3 + \frac{C_{\text{str}}^2}{16a^2}}} - \sqrt[3]{-\frac{C_{\text{str}}}{4a} + \sqrt{\left(\frac{\rho}{3}\right)^3 + \frac{C_{\text{str}}^2}{16a^2}}} \right)^2 - \frac{1}{\delta^2} \right] a,$$

where

$$\rho = \frac{C_{\text{str}}\delta^2(\delta - 2u) - 2a}{2a\delta^2}.$$

To simplify $F(u)$ we expand it in powers u in the vicinity of the point $u = 0$. Then, disregarding terms of order $O(u^3)$, we obtain

$$F(u) = a_1u + a_2u^2.$$

Having analyzed the function $F_i = F(u_i)$, where $i = \overline{1, 100}$, in the vicinity of the point $u = 0$ and using the least-squares method, we have

$$\sum 2(a_1u_1 + a_2u_1^2 - F_i)u_i = 0, \quad \sum 2(a_1u_i + a_2u_i^2 - F_i)u_i^2 = 0,$$

$$a_1 \sum u_i^2 + a_2 \sum u_i^3 = \sum F_i u_i, \quad a_1 \sum u_i^3 + a_2 \sum u_i^4 = \sum F_i u_i^2,$$

$$\Delta = \begin{vmatrix} \sum u_i^2 & \sum u_i^3 \\ \sum u_i^3 & \sum u_i^4 \end{vmatrix} = \left(\sum u_i^2 \right) \left(\sum u_i^4 \right) - \left(\sum u_i^3 \right)^2, \quad \Delta_1 = \begin{vmatrix} \sum F_i u_i & \sum u_i^3 \\ \sum F_i u_i^2 & \sum u_i^4 \end{vmatrix} = \sum F_i u_i \sum u_i^4 - \sum F_i u_i^2 \sum u_i^3,$$

$$\Delta_2 = \begin{vmatrix} \sum u_i^2 & \sum F_i u_i \\ \sum u_i^3 & \sum F_i u_i^2 \end{vmatrix} = \sum u_i^2 \sum F_i u_i^2 - \sum u_i^3 \sum F_i u_i, \quad a_1 = \frac{\Delta_1}{\Delta}, \quad a_2 = \frac{\Delta_2}{\Delta},$$

then $F(u) = 4.49 \cdot 10^8 u + 5.39 \cdot 10^{12} u^2$.

We denote the angles θ by $\theta_1, \theta_2,$ and θ_3 for all the shells of the three-shell support and determine them as the angles between the axis of the shells and the direction of the vector of the overall radial load according to the scheme in Fig. 2b.

We compute the cosines and sines of the angles θ_1 , θ_2 , and θ_3 for support 1 (Fig. 2b), expressing them by the known angles α_0 and α . This procedure is necessary for further solution of the equations of the model

$$\cos \theta_1 = \frac{1}{e} (x \cos \alpha_0 + y \sin \alpha_0), \quad (4)$$

$$\sin \theta_1 = \frac{1}{e} (x \sin \alpha_0 - y \cos \alpha_0), \quad (5)$$

$$\cos \theta_2 = -\frac{1}{e} [x \cos (\alpha_0 - 60^\circ) + y \sin (\alpha_0 - 60^\circ)], \quad (6)$$

$$\cos \theta_3 = -\frac{1}{e} [x \cos (\alpha_0 - 60^\circ) - y \sin (\alpha_0 - 60^\circ)]. \quad (7)$$

Expressions (4)–(7) also hold for support 2, where the angle θ corresponds to the angle ξ and θ_1 , θ_2 , and θ_3 correspond to ξ_1 , ξ_2 , and ξ_3 . The values of the clearance openings are determined as follows:

for the first support

$$u_1 = e \cos \theta_1, \quad u_2 = e \cos \theta_2, \quad u_3 = e \cos \theta_3;$$

for the second support

$$v_1 = e \cos \xi_1, \quad v_2 = e \cos \xi_2, \quad v_3 = e \cos \xi_3.$$

With account for expressions (4)–(7) the values of the clearance openings can be represented as

$$u_1 = x_1 \cos \alpha_0 + y_1 \sin \alpha_0, \quad u_2 = -x_1 \cos (\alpha_0 - 60^\circ) - y_1 \sin (\alpha_0 - 60^\circ),$$

$$u_3 = -x_1 \cos (\alpha_0 + 60^\circ) - y_1 \sin (\alpha_0 + 60^\circ).$$

Considering the scheme of the rigidity distribution in the support, we obtain

$$F_{1x} = F(u_1) \cos \alpha_0 + F(u_2) \cos (\alpha_0 + 120^\circ) + F(u_3) \cos (\alpha_0 + 240^\circ),$$

$$F_{1y} = F(u_1) \sin \alpha_0 + F(u_2) \sin (\alpha_0 + 120^\circ) + F(u_3) \sin (\alpha_0 + 240^\circ).$$

The rigidities for the second support F_{2x} and F_{2y} are computed analogously.

The damping in the supports is equal to $H_{1x} = h\dot{x}_1$, $H_{2x} = h\dot{x}_2$, $H_{1y} = h\dot{y}_1$, and $H_{2y} = h\dot{y}_2$.

To solve the equations of the mathematical model we introduce the notation of the variables

$$y_1 = x_3, \quad y_2 = x_4, \quad \dot{x}_1 = x_5, \quad \dot{x}_2 = x_6, \quad \dot{y}_1 = x_7, \quad \dot{y}_2 = x_8. \quad (8)$$

We also employ the notation

$$A_x = \frac{l}{M} [\omega^2 (m_1 \varepsilon_1 \cos \omega t + m_2 \varepsilon_2 \cos (\omega t + \gamma)) - R \cos \varphi_0 - F_{1x} - F_{2x} - H_{1x} - H_{2x}],$$

$$A_y = \frac{l}{M} [\omega^2 (m_1 \varepsilon_1 \sin \omega t + m_2 \varepsilon_2 \sin (\omega t + \gamma)) - R \sin \varphi_0 - G - N_2 - F_{1y} - F_{2y} - H_{1y} - H_{2y}],$$

$$B_x = \frac{l}{I_e} \left[\omega^2 (m_1 \varepsilon_1 s_1 \cos \omega t + m_2 \varepsilon_2 s_2 \cos (\omega t + \gamma)) - F \sin \varphi_0 l_3 - F_{1x} l_1 - F_{2x} l_2 - H_{1x} l_1 - H_{2x} l_2 + \frac{I_p \omega}{l} (x_7 - x_8) \right], \quad (9)$$

$$B_y = \frac{l}{I_e} \left[\omega^2 (m_1 \varepsilon_1 s_1 \sin \omega t - m_2 \varepsilon_2 s_2 \sin (\omega t + \gamma)) - F \sin \varphi_0 l_3 - N_2 l_4 - F_{1y} l_1 - F_{2y} l_2 - H_{1y} l_1 - H_{2y} l_2 + \frac{I_p \omega}{l} (x_5 - x_6) \right].$$

With account for (8) and (9), we write (2) in the form

$$A_x = \dot{x}_5 l_2 - \dot{x}_6 l_1, \quad A_y = \dot{x}_7 l_2 - \dot{x}_8 l_1, \quad B_x = \dot{x}_6 - \dot{x}_5, \quad B_y = \dot{x}_8 - \dot{x}_7.$$

Having solved these equations for x_5 , x_6 , x_7 , and x_8 , we obtain

$$\dot{x}_5 = \frac{1}{l} (A_x + l_1 B_x), \quad \dot{x}_6 = \frac{1}{l} (A_x + l_2 B_x), \quad \dot{x}_7 = \frac{1}{l} (A_y + l_1 B_y), \quad \dot{x}_8 = \frac{1}{l} (A_y + l_2 B_y);$$

$$\dot{x}_1 = x_5, \quad \dot{x}_2 = x_6, \quad \dot{x}_3 = x_7, \quad \dot{x}_4 = x_8. \quad (10)$$

Having substituted (8) and (9) into (10), we obtain the solution of the equation of motion of the rotor shaft in hydrodynamic supports in the form of a set of coordinates of the center of the end rotor surface at the instant of time t_{inst} forming the set of positions of the center of the end rotor surface over the period T .

Thus, the region of states represents the set of points each of which determines the instantaneous position of the center of the end rotor surface. The region of state is formed by simulation modeling of varied parameters in space with the use of uniformly distributed sequences [5, 6]. The parameter space is represented by a four-dimensional hyperparallelepiped bounded by the range of variation of each of the parameters: rotational velocity of the rotor, diametral clearance, viscosity of oil, and radial force.

It has been established that the dimensions of the region of states most strongly depend on the diametral running clearance in the hydrodynamic plain bearing. We have proposed the structure of a plain bearing with automatic adjustment of the clearance. It allows setting of the clearance in two steps (setting of the mounting clearance on the inoperative machine tool and fine adjustment at no-load) and control of the vibroactivity of the support in the process of operation of a rotor unit due to the use of a magnetorheological liquid and an elastic element in the form of a system of sylphon bellows of different diameters [7].

Based on the results of theoretical investigations, we have also developed a procedure of assembly of hydrodynamic supports and determination of the clearance in them [8]. The employment of the procedure at the Vistan Machine-Tool Plant (Vitebsk) made it possible to improve the precision of assembling the hydrodynamic supports of the spindles of grinding machines, which had a positive effect on the precision longevity of spindle units.

CONCLUSIONS

1. We have proposed a mathematical model of functioning of a rotor system with hydrodynamic plain bearings with allowance for the actually existing phenomenon of circular anisotropy of the rigidity of three-shell film lubrication bearings.

2. The operation has been modeled on the basis of a mathematical model developed with the use of uniformly distributed sequences, which has enabled us to obtain the region of states of the rotor system in the form of a set of points — a combination of the positions of the center of the end rotor surface.

3. Based on the investigations carried out, we have developed a procedure for determination of the clearance in the hydrodynamic three-shell plain bearing, whose adoption has made it possible to improve the precision of assembling the spindle units of grinding machines.

NOTATION

B_{sh} , size of the arc of the shell's working surface, m; c , rigidity of the support, N/m; C_{str} , constant structural rigidity, N/m; C_L , coefficient allowing for the location of the reference point along the shell length; C_{oil} , rigidity of the oil wedge, N/m; D_{sh} , bore diameter of the shells, m; e , shift of the center of the shaft from the initial position under the action of the resultant of external forces, m; \mathbf{F}_1 and \mathbf{F}_2 , forces produced by the rigidities of the first and second supports respectively, N; I_p , polar moment of inertia of the rotor, $\text{kg}\cdot\text{m}^2$; I_e , equatorial moment of inertia of the rotor, $\text{kg}\cdot\text{m}^2$; \mathbf{G} , weight of the wheel, kg; h_1 and h_2 , coefficients of damping in the first and second supports; \mathbf{H}_1 and \mathbf{H}_2 , damping forces in the first and second supports; $m_1\varepsilon_1$ and $m_2\varepsilon_2$, values of the disbalances equal to the unbalanced mass (m_1 and m_2) by the modulus of its eccentricity (ε_1 and ε_2), $\text{kg}\cdot\text{m}$; M , mass of the rotor, kg; l , distance between the supports, m; l_1 and l_2 , longitudinal coordinates of the first and second supports reckoned from the center of mass of the rotor, m; l_3 and l_4 , coordinates of the centers of mass of the wheel and the pulley reckoned from the center of mass of the rotor, m; L , length of the shell's working surface, m; n , number of revolutions of the shaft per minute; \mathbf{N}_2 , force applied to the driving pulley, N; \mathbf{R} , radial force, N; t , running time, sec; s_1 and s_2 , longitudinal coordinates of the planes in which the localized masses reckoned from the center of mass of the rotor are located, m; α_0 , angle between the OX axes and the shell axis, deg; α , angle between the OX axis and the vector of radial load, deg; δ , diametral clearance, m; γ , angle between the directions of the disbalances of masses I and II, deg; φ_0 , angle allowing for the direction of the radial force, deg; μ , oil viscosity, $\text{MPa}\cdot\text{sec}$; θ , coordinate of a point of the shell bearing relative to the plane of action of the resultant of external forces, deg; ω , rotational velocity of the rotor, sec^{-1} ; P , carrying capacity of the hydrodynamic bearing, N; q and ρ , coefficients of the cubic equation. Subscripts: sh, shell; oil, oil wedge; str, structural; p, polar; e, equatorial; L , shell length; inst, instantaneous; \cdot , first derivative; $\ddot{\cdot}$, second derivative.

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